Challenges of Existing in a Market as a Small, Low-quality Producer

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ABSTRACT. This paper explores the strategy of a small firm entering a monopolist's market thereby creating a duopoly market. The small firm avoids competing with the larger, incumbent firm by producing a lower-quality product at a lower price. The model here establishes an equilibrium under a specific set of assumptions and examines how exogenous factors affect prices, qualities and profits. Although the strategy might allow the firm to enter and earn a profit, the market conditions may make this position much less desirable to that of the large firm for several reasons. In the case explored here where tastes for the product are uniformly distributed, the small firm's profit is about six percent of that of the larger firm. The smaller firm is more severely threatened by the entrance of a third firm. Furthermore, even if the smaller firm can cut costs, its position is not well suited for exploiting such increases in efficiencies.

1. Introduction

This paper develops a theoretical model that describes a duopoly market equilibrium where a small firm has entered a monopolist's market with a product of lower quality than the pre-existing brand. Although the strategy of entering a market with a lower quality product has been successful in some markets, the results here depict a case where the entering small-firm will be at a clear disadvantage. The small firm will have a much lower profit level; furthermore, unilateral improvements in efficiency by the small firm will lead to only small increases in profits, and the small firm will experience a proportionally larger drop in profits than the large firm when a third firm enters the market.

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Many firms have successfully entered markets with lower quality products designed to capture a share of the market ignored by incumbent firms. In his landmark text *The Innovator's Dilemma*, Christensen (1997) notes that established firms often leave themselves vulnerable to competition from low-quality brands.

> Rational managers . . . can rarely build a cogent case for entering small, poorly defined low-end markets that offer only lower profitability. In fact, the prospects for growth and improved profitability in upmarket value networks often appear to be so much more attractive than the prospect of staying within the current value network, that it is not unusual to see well-managed companies leaving (or becoming uncompetitive with) their original customers as they search for customers at higher price points. (p. 77)

Using the disk-drive industry and the mechanical excavator industry as historical examples, Christensen describes how established firms have suffered by focusing on product improvement and increasing prices while ignoring a large segment of the market that would prefer to buy a lower-quality product at a lower price.

Henry Ford's introduction of the Model-T automobile and Charles Schwab's introduction of the discount broker concept serve as familiar examples where a firm has entered a market promoting its product as being lower in quality to existing brands but also lower in price. These firms then went on to become important competitors in their respective markets. Can the strategy succeed in all markets? Given the assumptions in this model, the low-quality producer can successfully enter the market, but the firm will be small and subordinate to the incumbent, high-quality producer.

In the model we assume that the product is not a major purchase for the consumers so there is no

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income effect associated with the purchase. Also, consumers make only one purchase of the product per period. These two assumptions concerning the relative size and frequency of the purchase could apply to markets for services like telephone and internet providers. Another very important assumption on the demand side is that each consumer is distinguished by a unique, quantifiable taste factor for the product. The taste factors follow a uniform distribution over a finite range. As for the supply side, each firm has a quadratic cost-of-quality function that includes the quality level of a free variety of the good. The cost-of-quality function also includes a cost factor that may have a unique value for each firm. Once a firm determines its quality level, per unit production costs of the good are zero.

The following discussion subsumes the main properties of the equilibrium given the assumptions. In this duopoly model, the prices maintain a constant ratio because they change by the same percentage in response to an absolute change in each exogenous variable. Firms change their quality levels equally in absolute terms to match a quality change in the free product. The large firm's quality level is more sensitive in absolute terms to changes in the model's cost factor and the size of the market. The model shows that the low-quality firm will sell to fewer customers, and therefore be "small." The profit level of the small firm is about six percent of that of the large firm while selling to half as many buyers. If a third firm chooses to enter with the low-quality strategy and produces the lowest quality of the three, the largest firm's profit falls more in absolute terms, but the one-time small and now intermediate firm's profit falls more in percentage terms.

Furthermore, the small firm is not well positioned to fully benefit from increases in efficiency. If the small firm unilaterally lowers the cost factor by a given percentage, its profit increases by a smaller percentage. As the small firm tries to increase market share and price, the large firm will react optimally by doing the same even when its cost factor has not changed. Interestingly enough, the model demonstrates that the small firm's profit will increase more if it shares with the large firm the innovation that had lowered its costs. If the cost factor for both firms are equal and they both lower the factors by an equal amount, the profits of both firms will increase by a percentage equal to the percentage decrease in the cost factors. Although the small firm benefits by sharing the efficiency innovation, the large firm benefits much more. A unilateral percentage decrease in the cost factor by the large firm increases its profit by a greater percentage, furthermore, and increases the profit of the small firm only slightly. The large firm will not benefit by sharing technology if it is the innovator. These results suggest that an innovating small firm would have an incentive to risk short-term losses and challenge the large firm directly for its dominant position.

When a small firm considers entering a market by producing a low-end product, it can look to this model to see how the targeted industry is similar to the theoretical model developed here and what challenges it might expect based upon those similarities. The review of the literature in the next section summarizes relevant research concerning the entrance of firms into markets and the game-theory models that describe how new entrants and incumbents interact. Section three introduces the basic parameters and shows the results of the equilibrium for a monopoly market. In section four the small firm enters, and the analysis there shows how the firms interact to determine equilibrium prices, qualities, and profits. Section five shows how changes in exogenous variables affect this equilibrium. Section six examines the effect of each firm unilaterally lowering its cost factor as well as the effect of a third firm entering the market. Section seven concludes.

2. Review of the literature

The basic tenet of game theory is that firms may find it beneficial to cooperate with respect to the nature of their competition. This paper examines one particular situation where an incumbent monopolist implicitly agrees to allow a small firm to enter its market as long as the small firm keeps its place as the producer of a low-quality good. By agreeing upon this strategy, the incumbent avoids setting up costly barriers to entry, and the small firm increases its probability of successfully making a profit albeit a relatively low one. The review of the literature in this section summarizes the support for a model using this scenario.

Most markets support more than one quality of a product, e.g., automobiles and appliances. With the deregulation of financial markets, investors can choose between full-service and discount brokers. Previous authors have built credible models that have explained the behavior of agents in markets where firms offer different quality levels. More quality levels improve social welfare, furthermore, because consumers have more choices; see Meade (1974), Leland (1977), Mussa and Rosen (1978), and Anderson et al. (1992).

The model in this paper examines a case where, initially, a monopolist produced a single quality of the product. A small firm then enters, and the entering firm produces and sells a product of lower quality at a lower price to a smaller share of the market. The analysis proceeds on the assumption that the monopolist cannot profitably deter the entrance of the second firm. Previous research that lends credibility to this assumption includes Dixit (1980), Schwartz and Thompson (1986) and Schwartz and Baumann (1987). The basic argument here is that the large firm's loss in profit from allowing the small, low-quality producer in is less than the costs incurred from keeping the small firm out of the market.

Many previous authors have examined the issue of quality-of-product and entering firms. Bain (1946), Schmalensee (1982) and Conrad (1983) show that a pioneering brand will have a stronger market position over firms who enter later because consumers discount the quality of the new variety of the good. If this were true, even if an entering firm produces a product of equal quality, consumers would perceive it to be of lower quality. In some cases entering firms have an incentive, therefore, to resign themselves to producing a lower quality product.

A smaller firm entering a monopolist's market with a good of lower quality will affect the behavior of the incumbent firm. Many previous authors have examined the issue of incumbent reaction with respect to various types of competition. In the horizontal competition introduced by Hotelling (1929), subsequent authors such as Jehiel (1992), Tirole (1988), Boyer and Moreaux (1987) and Stahl (1982) have developed scenarios where the firms cooperate by choosing positions that are optimal for the two firms once they agree to share the market. Empirical work in this area includes Hjorth-Andersen (1988), Shaw (1982) and Swann (1985).

Research into the area of quality competition predates Hotelling (1929). Bertrand (1883) demonstrates that if firms do not engage in quality differentiation and compete directly with price, then quality levels will rise, and prices will fall until prices equal marginal costs. Clearly firms have an incentive to cooperate, and one form of cooperation is for the firms to agree on the quality each will produce.

To model this behavior, authors have developed vertical competition models for both simultaneous and sequential entry. Pescott and Visscher (1977) and Hay (1976) pioneered the modern sequential-entry models. Lane (1980) extended the model to allow for endogenous prices which has characterized most later models including the one in this paper. Subsequent work in this area includes Bernheim (1984), Harris (1985), Eaton and Ware (1987), Dewatripont (1987), Benoit and Krishna (1987), Vives (1988), Mclean and Riordan (1989), and Anderson et al. (1992).

The model developed here borrows directly from Anderson et al. (1992), henceforth ADT. The next section summarizes the assumptions of the ADT model and the extensions and modifications introduced in this paper. Comparative statics illustrate how the two firms will react to changes in exogenous parameters. Under the proposed assumptions, the model shows the optimal behavior of the small firm and large firm when market conditions change.

3. The monopoly model

This section serves two purposes. It introduces the parameters of the market and the corresponding simplifying assumptions of the model. The model here uses most of the basic parameters in the ADT model, but the analysis here adds a free good and a specific cost function. This section also describes the equilibrium with a single seller under those assumptions. That equilibrium is a standard result, but it sets the stage for examining the effects of the smaller firm's entry, and section four examines those effects.

Although the model here relies heavily on the ADT model, it builds upon that model by introducing a free variation of the product. This free product may represent a variety of parameters. It could actually represent a free alternative to a service or good. In the case of Internet services, the free product may mean the potential buyer chooses not to go online and uses other means for gathering information. As libraries and other institutions offer the public free Internet access during specified hours, on the other hand, the potential buyer can consume via this free alternative. After its inclusion in the demand function below, which will determine the purchase/not purchase decision, the free good's quality also plays a role in the cost function.

There is not a specific cost function in the basic ADT model. The only parameters are consumer income denoted y, the price of product "i" denoted P_i, the quality of the product denoted q_i, and a taste factor for the particular consumer denoted θ. In the monopoly case, each period agents may purchase and consume a single unit of the product labeled "i". In the extension of the model developed here, the agent may choose to consume a free variety of the good where $i = F$ or purchase from the large firm, $i = L$. In either case, the net increase in utility from consuming the good depends on the price of the good and each agent's unique preference factor denoted θ.

ADT assumes the following indirect utility function that includes the net increase in utility from choosing to consume a particular variety of the product:

$$V_i(\theta) \equiv y - P_i + \theta \times q_i, \quad i = 1 \ldots n. \tag{1}$$

The net benefit of consuming product variety "i" is given by $\theta \times q_i - P_i$ or simply $\theta \times q_F$ if the consumer chooses the free good. In a single seller case the consumers have the choice between q_L and q_F. The subscript F represents the generalization of a free product added to the ADT model, and q_F represents the quality of that free product. The symbol q_L represents the quality level offered by the monopolist in the single-seller case, and the subscript L refers to this monopolist being the larger firm in the duopoly. When the smaller firm enters the market, it offers quality level q_S. The following relationships hold: $q_F < q_S < q_L$.

The range of preference factors that satisfy $\theta_u \geq \theta \geq \theta^* = P_L/(q_L - q_F)$ characterizes buyers of the monopolist's product. The term θ_u is the upper limit of tastes in the population of buyers. Substituting this into the profit function and taking the first derivative yields the optimal price, and that price is $P_L^* = \theta_u \times (q_L - q_F)/2$. How many consumers are included in the range specified by $\theta_u \geq \theta \geq \theta^*$? To answer this question requires assumptions about the distribution of θ. As in ADT, the model here assumes that tastes are uniformly distributed: $\theta \sim \text{Uniform}[\underline{\theta}, \theta_u]$ where it may be the case that $\underline{\theta} = 0$. As will be discussed later, the results of the model and the success of a firm using the low-quality entry strategy will hinge upon assumptions concerning the distribution of tastes.

Assuming a uniform distribution of θ over $[0, \theta_u]$, P_L^* is half the bid-price of the buyer characterized by θ_u. It follows that the marginal preference is

$$\theta^* = \frac{\theta_u \times (q_L - q_F)/2}{(q_L - q_F)} = \frac{\theta_u}{2}.$$

This is a standard result in a monopoly market where the firm has zero per-unit-costs of production. It is immutable with respect to price, quality, and the associated quality cost-function. For any interior solution, quality and price do not affect this outcome.

Determining profit, however, requires a specific cost function. It is likely that the firms would face increasing costs to quality improvement. Furthermore, the existing available technology would affect the costs. The chosen cost function uses a quadratic form to incorporate the increasing costs. It also incorporates two technology factors represented by the quality of the free alternative and a scale parameter denoted ξ. The cost function is,

$$C(q_i) = \xi \times (q_i - q_F)^2.$$

Some explanation for using the free good as a general level of technology is in order. As technology improves, it is reasoned, the available free alternatives would increase in quality. The market for cable television might serve as a good example. A consumer can choose between subscribing to a cable network or using the free alternative available on the airwaves. Increases in the availability and quality of free programming reflect the state of technology in the telecommunications industry. This level of technology would influence the costs of providing cable programming. Using the case of the Internet as another example, as internet technology increases so does the availability of costless information from other sources. That is why the expression $(q_i - q_F)$ is a component of the cost function.

Although the model assumes a specific cost function, future research can explore the use of other cost functions. Furthermore as mentioned earlier, practitioners can modify the model to better reflect the actual conditions faced. The cost function developed here provides a good, basic function to build upon.

Using the demand and cost functions, the expressions for optimal price and market segment yields the following profit function:

$$\pi = \frac{\theta_u^2}{4} \times (q_L - q_F) - \xi \times (q_L - q_F)^2.$$

The optimal increase in quality above the free level is determined by the first-order condition[1] $q_L = 0.125 \times \theta_u^2/\xi + q_F$. The signs of the relevant derivatives are unambiguous: $\mathrm{d}q_L/\mathrm{d}\xi = -0.125 \times \theta_u^2/\xi^2 < 0$, and $\mathrm{d}q_L/\mathrm{d}q_F = +1$. Changes in cost and the quality level of the free good induce changes with signs that appeal to intuition.

The results $0 < \mathrm{d}q_L/\mathrm{d}q_F$ and $\mathrm{d}q_L/\mathrm{d}\xi < 0$ hold when θ follows a distribution other than uniform. A cumulative distribution function, denoted $\xi(\cdot)$, can help create a more general expression for profit in the ADT case examined here:

$$\pi = P_L \times \left[1 - \phi\left(\frac{P_L}{q_L - q_F}\right)\right] - \xi \times (q_L - q_F)^2.$$

Optimal price, given θ^*, is $P_L^* = [1 - \phi(\theta^*)]/[(q_L - q_F) \times \phi'(\theta^*)]$. Substituting P_L^* into $\theta^* = P_L/(q_L - q_F)$ gives the following expression: $\theta^* = [1 - \phi(\theta^*)]/\phi'(\theta^*)$, and θ^* is a constant nominal value for many distributions. Unique solutions exist for θ^* and the second-order conditions are satisfied when $\phi(\cdot)$ represents a uniform, normal or exponential distribution.[2] The existence of a unique and constant value for θ^* is sufficient to give $0 < \mathrm{d}q_L/\mathrm{d}q_F$ as well as $\mathrm{d}q_L/\mathrm{d}\xi < 0$ in this model.

Different distributions will give different levels of profit. In the case of a uniform distribution, the equilibrium conditions will yield a profit of $(\theta_u^2/\xi) \times (0.015625)$ for the monopolist. If the distribution of tastes is $\phi(\theta) = 2 \times \theta - \theta^2$, where $0 < \theta < 1$, then the unique value for marginal taste is $\theta^* = \theta_u \times 0.333333$ and the seller sells to 0.444444 of the market. The profit is $(\theta_u^2/\xi) \times (0.012346)$ for the monopolist in this case.

The results developed in this section are straightforward. The purpose here has been to demonstrate the model in the relatively simple monopolist case. The next section investigates the results when a second, smaller firm enters the market with a product of lower quality.

4. A smaller firm enters the monopolist's market

To successfully coexist in a market, firms may develop a method to share the market to lower competitive pressures. This section describes the equilibrium when the method of sharing the market is for one firm to be comparatively small and produce a lower-quality product. In this model, the small firm's improvement over the available free-alternative is about one-fifth that offered by large firm, and its price is about one-tenth of that charged by the large firm. The small firm produces one third of the industry output but earns less than six percent of the industry economic profits, and industry profits are lower in the duopoly than in the monopoly case.

The sharing of the market by the two firms is better for the firms than some behavioral alternatives. If a second identical firm enters a market dominated by a single firm, price and quality competition could drive quality levels up and prices down. Indeed, if production costs are zero, as assumed here, then one equilibrium could be $P_S^* = P_L^* = 0$ and the qualities become equal at some upper limit. "This equilibrium can be viewed as an extension of Bertrand's[3] result for homogeneous products." [ADT p. 306] Hence, when a second firm enters a market, in order to survive it may choose to differentiate itself with respect to quality. The monopolist may decide that the low cost solution is to allow the small firm to enter with the lower quality product as opposed to fighting the entrance of the firm.

As the small firm prepares to enter the market, the large firm will plan a reaction that the small firm must anticipate. Following ADT, we assume that firms can adjust prices more easily than qualities; therefore, the firms engage in a quality-then-price two-stage game.[4] In the first stage of the game, firms select qualities. Each firm knows the reaction function of the other and the corresponding equilibrium prices.

With two firms selling products where $(q_S, P_S) << (q_L, P_L)$, then a consumer purchases the higher quality if

$$V(\theta_S) \equiv y - P_S + \theta_S \times q_S < V(\theta_L) \equiv y - P_L + \theta_L \times q_L.$$

An indifferent agent has a value for the preference parameter defined by $\theta^* = (P_L - P_S)/(q_L - q_S)$. This value defines the market segment for each of the two firms. In the ADT model, an agent with a preference factor that falls within the following interval, $\underline{\theta} < \theta < \theta^*$, purchases q_S.

Each firm prepares for the second stage of the game by entering θ^* into its respective profit function and solving for the optimal price: $P_i^* = f(P_j, q_S, q_L, \underline{\theta}, \theta_u)$. In the ADT model, by assumption, cost is solely a function of quality such that per-unit production costs of the good are zero. If the solutions yield $0 < P_i < y$, then optimal prices are[5]

$$P_L^* = \frac{(2 \times \theta_u - \underline{\theta}) \times (q_L - q_S)}{3},$$
$$P_S^* = \frac{(\theta_u - 2 \times \underline{\theta}) \times (q_L - q_S)}{3}.$$

Prima facie, the reader may question why these solutions for prices imply that $dP_S/dq_L > 0$ and $dP_S/dq_S < 0$. As the following analysis will show, exogenous factors such as the range of preferences ultimately determine the distance $(q_L - q_S)$. When the range of preferences expands, as section five will show, the large firm's quality will increase more than the small firm's quality, i.e., $d(q_L - q_S)/d\theta_u > 0$ which gives $dP_L^*/d\theta_u > 0$ and $dP_S^*/d\theta_u > 0$. That is why an increase in the distance $(q_L - q_S)$ is associated with an increase in the price of both firms.

The distance $(q_L - q_S)$ appears in the profit functions of both firms, and those profit functions will eventually reduce to functions of exogenous parameters. Incorporating the expressions for optimal prices into the profit functions in the first stage of the game yields the following expressions for the profit of each firm,

$$\pi_L(q_S, q_L) = \frac{(2 \times \theta_u - \underline{\theta})^2 \times (q_L - q_S)}{9} - C(q_L),$$
$$\pi_S(q_S, q_L) = \frac{(\theta_u - 2 \times \underline{\theta})^2 \times (q_L - q_S)}{9} - C(q_S).$$

In the special case of zero costs, i.e., $C(q_i) = 0$ for all $0 \leq q_i$, then the two producers space themselves on opposite ends of the consumer preference spectrum: $q_L = \overline{q}$ and $q_S = \underline{q}$. This softens price competition. If $0 < C(q_i)$, then whether the solutions are interior to $[\underline{q}, \overline{q}]$ will depend upon that cost function. The variation of the ADT duopoly model developed here uses the cost function introduced in the previous section.

As in the ADT model, we continue to assume a uniform distribution of consumer preferences, and to simplify the analysis we assume $\underline{\theta} = 0$. In the first stage of the game, firms recognize how their choices of q_i determine market shares. The preference factors for the marginal buyers are

$$\theta_L^* = \frac{P_L - P_S}{q_L - q_S}, \quad \theta_S^* = \frac{P_S}{q_S - q_F}. \tag{2}$$

In our model, the length $\theta_u - \theta_L^*$ indicates the number of buyers of firm L's product. The corresponding measure for firm S is $\theta_L^* - \theta_S^*$. Solving for each P_i^* as before yields

$$P_L^* = \frac{2 \times \theta_u \times (q_L - q_S) \times (q_L - q_F)}{4 \times q_L - q_S - 3 \times q_F},$$
$$P_S^* = \frac{\theta_u \times (q_S - q_F) \times (q_L - q_S)}{4 \times q_L - q_S - 3 \times q_F}.$$

For ease of notation we write $P_L^* = 2 \times \theta_u \times \delta_{L,S} \times \delta_{L,F}/\omega$ and $P_S^* = \theta_u \times \delta_{S,F} \times \delta_{L,S}/\omega$. The implied transformations are $\delta_{L,F} = q_L - q_F$, $\delta_{S,F} = q_S - q_F$, $\delta_{L,S} = q_L - q_S$, $\omega = 4 \times q_L - q_S - 3 \times q_F$

The profit functions in the first stage of the game are

$$\pi_L = \frac{4 \times \theta_u^2 \times \delta_{L,S} \times \delta_{L,F}^2}{\omega^2} - C(q_L),$$
$$\pi_S = \frac{\theta_u^2 \times \delta_{S,F} \times \delta_{L,S} \times \delta_{L,F}}{\omega^2} - C(q_S).$$

Recalling the cost function in section three, a profit advantage exists for firm L if the cost function is $C(q_i) = \xi \times (q_i - q_F)^2$. The first-order conditions for firms L and S respectively are[6]

$$\xi = \frac{2 \cdot \theta_u^2 \cdot (2 \cdot \delta_{S,F}^2 - 3 \cdot \delta_{S,F} \cdot \delta_{L,F} - 4 \cdot \delta_{S,F}^2)}{\omega^3},$$

$$\xi = \frac{\theta_u^2 \times \delta_{L,F}^2 \times (4 \times \delta_{L,F} - 7 \times \delta_{S,F})}{2 \times \delta_{S,F} \times \omega^3}.$$

Equating the two expressions for ξ gives a third degree polynomial in a variable defined as $\delta_{L,F}/\delta_{S,F}$. Only a single real root exists: $\delta_{L,F}/\delta_{S,F} = 5.25123$. Entering this value into the equations above yields solutions for the quality levels,

$$q_L = \frac{0.12665 \times \theta_u^2}{\xi} + q_F,$$

$$q_S = \frac{0.02412 \times \theta_u^2}{\xi} + q_F.$$

These expressions define relationships between the produced levels of quality, q_S and q_L, and the exogenous variables θ_u^2, q_F and ξ. Using the ratio θ_u^2/ξ as a unit of measure, the smaller firm must plan to offer a quality level above q_F that is less than one-fifth the distance of $q_L - q_F$:

$$\frac{(q_L - q_F)}{(q_S - q_F)} = 5.25123.$$

Our extension of the ADT model yields other general results that do not require specific assumptions concerning parameter inputs. If both firms are producing at their optimally-spaced quality levels, their market shares are invariant in this model. Interestingly, the competition forces the large firm to increase its market share from 50% to 52.5% of all buyers. The small firm sells to 26.25% of all buyers. These market shares are immutable as long as both sellers remain in the market and the cost functions are identical. Profits per period are $\pi_L = (0.012666) \times \theta_u^4/\xi$ and $\pi_S = (0.000767) \times \theta_u^4/\xi$. Firm L clearly has a superior position.

These results would prove useful in a pro-competition argument for a given industry. Economic profits for the industry fall from $(0.015625) \times \theta_u^4/\xi$ to $(0.013433) \times \theta_u^4/\xi$. In the monopoly market, the single seller sold to half the market with a quality level of $q_L = (0.125 \times \theta_u^2)/\xi + q_F$. In the two-seller market, 52.5% purchase a good of a higher quality. Furthermore, an additional 26.25% of the consumers can now purchase the good at an affordable price although with a lower quality. Table I contains the results of the monopoly and duopoly model for the prices, qualities, profits and market shares.

The reader must keep in mind that these results depend upon the assumptions of the model. One of the more important assumptions is the uniform distribution of tastes. As illustrated in the monopolist case, equilibrium is possible with other distributions, and it will change the equilibrium results. A distribution like $\phi(\theta) = 2 \times \theta - \theta^2$ explored in the monopolist case might give the lower quality producer more of an edge as the distribution is right skewed. Such a right skewed distribution may explain the success of the introduction of basic products in some markets and not in others. Unfortunately, the mathematical analysis dramatically increases in complexity when other distributions are introduced into the duopoly case. Future research will address that topic.

This section has summarized the basic quality

TABLE I
Comparison of monopoly and doupoly results

Variable	Monopoly	Doupoly: Large firm	Doupoly: Small firm
Prices	$P_L = \frac{0.06250 \times \theta_u^3}{\xi}$	$P_L = \frac{0.05383 \times \theta_u^3}{\xi}$	$P_S = \frac{0.00513 \times \theta_u^3}{\xi}$
Qualities	$q_L = \frac{0.12500 \times \theta_u^2}{\xi} + q_F$	$q_L = \frac{0.12665 \times \theta_u^2}{\xi} + q_F$	$q_S = \frac{0.02412 \times \theta_u^2}{\xi} + q_F$
Profits	$\pi_L = \frac{0.015625 \times \theta_u^4}{\xi}$	$\pi_L = \frac{0.012666 \times \theta_u^4}{\xi}$	$\pi_S = \frac{0.000767 \times \theta_u^4}{\xi}$
Market shares	50% of potential buyers	52.5% of potential buyers	26.25% of potential buyers

competition model in a duopoly market with a specific distribution of preferences and cost function. Future research can build upon this model just as this model is built upon that proposed by ADT by introducing a free variety of the product and assuming a specific cost function. Under these conditions, a unique equilibrium exists. The next section examines how changes in exogenous parameters affect that equilibrium. The final section before the conclusion illustrates how the small firm has an unfavorable position in that it cannot exploit increases in efficiencies, and the small firm bears a larger proportional loss in profit when a third firm enters producing an even lower quality product.

5. Small and large firm reactions to exogenous shifts

This section examines how the firms react to changes in the quality of the free good, the cost parameter and the range of preferences. Comparative statics yield interesting results with respect to how the equilibrium quality levels and prices move with respect to each other. The results show that the large firm and small firm adjust quality and prices in the same proportions in reaction to changes in cost and market size. The parameter denoted q_F does not affect prices and profits, but q_F affects each quality level in equal absolute amounts.

Recalling the expressions for the quality levels in section four,

$$q_L = \frac{0.12665 \times \theta_u^2}{\xi} + q_F,$$

$$q_S = \frac{0.02412 \times \theta_u^2}{\xi} + q_F;$$

we observe that the $dq_L/dq_F = dq_S/dq_F = 1$. The two firms vary their absolute quality levels equally in response to changes in the free quality level. The reader may recall that we saw $dq_L/dq_F = 1$ in the monopoly case as well. Furthermore, the absolute distance between each quality level and the free quality is determined entirely by the exogenous parameters θ_u and ξ. The solutions for quality levels show that q_L is more sensitive than q_S to shifts in θ_u and ξ,

$$\frac{dq_L}{d\theta_u} = \frac{0.25330 \times \theta_u}{\xi} > \frac{dq_S}{d\theta_u} = \frac{0.04824 \times \theta_u}{\xi}$$

$$\left|\frac{dq_L}{d\xi}\right| = |-(0.12665 \times \theta_u^2) \times \xi^{-2}| >$$

$$\left|\frac{dq_S}{d\xi}\right| = |-(0.02412 \times \theta_u^2) \times \xi^{-2}|.$$

In each case, the derivative of the large firm is larger by a factor of about 5.25.

This has important implications with respect to the prices. First, we observe that this means the prices change by the same percent in response to a given absolute change in all the exogenous variables. We recall from the previous section:

$$P_L^* = \frac{2 \times \theta_u \times (q_L - q_S) \times (q_L - q_F)}{4 \times q_L - q_S - 3 \times q_F},$$

$$P_S^* = \frac{\theta_u \times (q_S - q_F) \times (q_L - q_S)}{4 \times q_L - q_S - 3 \times q_F}.$$

Clearly $P_L^*/P_S^* = 2 \times (q_L - q_F)/(q_S - q_F) = 2 \times 0.12665/0.02412 \approx 10.5$. Furthermore, the price elasticity with respect to q_F is zero. Similarly, the solutions for profits in section four are $\pi_L = (0.012666) \times \theta_u^4/\xi$ and $\pi_S = (0.000767) \times \theta_u^4/\xi$. Thus, profits are not affected by q_F either.

When the solutions for equilibrium qualities are substituted into the price functions, they become:

$$P_L = \frac{0.05383 \times \theta_u^3}{\xi},$$

$$P_S = \frac{0.00513 \times \theta_u^3}{\xi}.$$

The price of the large firm is more responsive in absolute terms to shifts in the parameters θ_u and ξ which was implied by $P_L^*/P_S \approx 10.5$. Table II summarizes the effects of exogenous parameter shifts on prices, qualities and profits. Table II also lists the relationships between the pairs of prices, qualities and profits.

As with any model, these results clearly depend upon the imposed assumptions. After the next section that explores the effects of endogenous shocks, concluding remarks in section seven address possible effects of relaxing some of the assumptions of the model.

TABLE II
Effects of shifts in exogenous parameters

Variable	Relationship	Effect of Δq_F	Effect of $\Delta\xi$	Effect of $\Delta\theta_u$
Prices	$P_L = 10.502 \times P_S$	No effect	Negative: $\lvert\Delta P_L\rvert > \lvert\Delta P_S\rvert$	Positive: $\Delta P_L > \Delta P_S$
Qualities	$q_L - q_F = 5.251 \times (q_S - q_F)$	Positive: $\Delta q_L = \Delta q_S$	Negative: $\lvert\Delta q_L\rvert > \lvert\Delta q_S\rvert$	Positive: $\Delta q_L > \Delta q_S$
Profits	$\pi_L = 16.514 \times \pi_S$	No effect	Negative: $\lvert\Delta\pi_L\rvert > \lvert\Delta\pi_S\rvert$	Positive: $\Delta\pi_L > \Delta\pi_S$

6. Endogenous shocks

This section explores the effect of two endogenous factors. First, this section explores how much the small firm can hope to improve its profit from a unilateral increase in efficiency as embodied in a lowering of the term ξ. Second, this section determines the effects on profits when a yet smaller firm enters the market. The results suggest that the small firm in the duopoly has a much less favorable position. The small firm's position does not encourage increases in efficiency, and its profit falls more in percentage terms when the third firm enters.

In the duopoly case, as mentioned earlier, the large and small firm profits are respectively $\pi_L = (0.012666) \times \theta_u^4/\xi$ and $\pi_S = (0.000767) \times \theta_u^4/\xi$. The small firm earns about 6% of what the large firm does in profit. When the small firm enters the incumbent's market, knowing this, it may hope to increase future profits with improved means of production. If the small firm improves efficiency and does not use these improvements to directly challenge the large firm, the increases in profit will be fairly small. Table III lists the profits of the small and large firm when the small firm has a lower value of ξ. Applying the appropriate subscripts to ξ, Table III lists results for selected values of $\xi_S/\xi_L < 1$.

Table III illustrates the plight of the small firm when trying to innovate by lowering costs of production. When the small firm lowers costs and attempts to sell to a larger segment of the market at the low end of the taste market, the large firm will expand also. The expansion of the large firm will hurt its profit and diminish the gains of the

TABLE III
Effects of the small firm improving efficiency by unilaterally lowering ξ_S

$\xi_S/\xi_L =$	Market of L	Market of S	π_L	π_S	$\pi_L + \pi_S$
1	0.525	0.2625	$\frac{0.012666 \times \theta_u^4}{\xi_L}$	$\frac{0.000767 \times \theta_u^4}{\xi_L}$	$\frac{0.013433 \times \theta_u^4}{\xi_L}$
0.5	0.540	0.270	$\frac{0.009852 \times \theta_u^4}{\xi_L}$	$\frac{0.001232 \times \theta_u^4}{\xi_L}$	$\frac{0.011084 \times \theta_u^4}{\xi_L}$
0.25	0.555	0.2775	$\frac{0.007037 \times \theta_u^4}{\xi_L}$	$\frac{0.001760 \times \theta_u^4}{\xi_L}$	$\frac{0.008800 \times \theta_u^4}{\xi_L}$
0.10	0.570	0.285	$\frac{0.003782 \times \theta_u^4}{\xi_L}$	$\frac{0.002276 \times \theta_u^4}{\xi_L}$	$\frac{0.006058 \times \theta_u^4}{\xi_L}$
0.01	0.580	0.290	$\frac{0.000782 \times \theta_u^4}{\xi_L}$	$\frac{0.002952 \times \theta_u^4}{\xi_L}$	$\frac{0.003734 \times \theta_u^4}{\xi_L}$
0.0001	0.582	0.291	$\frac{0.000284 \times \theta_u^4}{\xi_L}$	$\frac{0.003020 \times \theta_u^4}{\xi_L}$	$\frac{0.003304 \times \theta_u^4}{\xi_L}$

small firm. When $\xi_S/\xi_L = 0.5$, for example, the small firm's profits increase by a factor of 0.6063. The profit of the large firm diminishes by a factor of 0.2222 compared to when $\xi_S = \xi_L$.

In the monopoly model, a decrease of ξ by one-half would double profits for the innovating firm. Similarly, in the duopoly, if $\xi_S = \xi_L$ and both cost factors decrease by one-half then the profits of both firms will double. Interestingly enough, the competitive pressures in the duopoly market are so strong that the small firm is better off sharing any innovation that improves efficiency. In summary, when the small firm tries to increase quality and market share from a unilateral drop in ξ_S, the large firm will react by cutting its prices and increasing its market share despite ξ_L being unchanged. The small firm will be able to increase its profits, but by a lower percentage than that associated with the fall in ξ_S.

If the small firm shares the technology that lowered ξ_S to say ξ_S' and so now $\xi_S' = \xi_L'$; then both firms will experience an equal percentage increase in profits that is equal to the percentage decrease of the cost factor. In the case $\xi_S' = 0.5 \times \xi_S$ and allowing ξ_L to fall by the same percentage, profits of the small firm will now be $2 \times 0.000767 \times \theta_u^4/\xi_{\text{old}} = 0.001534 \times \theta_u^4/\xi_{\text{old}}$, where ξ_{old} represents the old cost factor for both firms before the increase in efficiency. The value $0.001534 \times \theta_u^4/\xi_{\text{old}}$ is greater than the value $0.001232 \times \theta_u^4/\xi_{\text{old}}$ in the row-two, column-five cell in Table III. This inequality illustrates the gains for the small firm for sharing its methods for lowering costs. The large firm gains more from the fall in the cost parameter, however, as the shared innovation allows its profit to double from $0.012666 \times \theta_u^4/\xi_{\text{old}}$ to $0.025332 \times \theta_u^4/\xi_{\text{old}}$.

As a short digression, if the large firm is the innovator and unilaterally reduces ξ by one half, market shares will contract slightly to 0.514 and 0.257 for the large and small firm respectively. The profit of the large firm will more than double to $0.027572 \times \theta_u^4/\xi_{\text{old}}$, and the profits of the small firm will rise to $0.000861 \times \theta_u^4/\xi_{\text{old}} = 0.000861 \times \theta_u^4/\xi_s$. The result for the large firm approaches but is still lower than the corresponding result for an equal final value of ξ in the case of the monopoly. Also, the large firm does not have an incentive to share its technology with the small firm. If it did share, the profits would then be $0.025332 \times \theta_u^4/\xi_{\text{old}}$, and $0.001534 \times \theta_u^4/\xi_{\text{old}}$ for the large and small firm respectively.

The results suggest that as long as the large firm follows the assumed behavior, the small firm will benefit slightly from the large firm's unilateral improvements in efficiency. However, an innovating large firm could also be more likely to abrogate the agreement and engage in short-run predatory-pricing to drive the small firm from the market. Furthermore, an innovating small firm has more incentive for such behavior when it can lower costs unilaterally. The results on Table III show that even when $\xi_S = 0.25 \times \xi_L$, the large firm's profit is higher than the small firm's profit.

The last column of Table III illustrates how unilateral increases in efficiency by the small firm actually hurt industry profits. Industry profits decline as the small firm lowers ξ_S. As ξ_S falls unilaterally, at some point both firms benefit from a switch in positions. This is clearly seen by the fact when $\xi_S/\xi_L = 0.01$, the profits of the large firm are approximately equal to that of the small firm before the fall in ξ_S. Determining the exact solution for ξ_S/ξ_L would be too much of a digression. The fact that a switch of a position at some point can benefit both firms is important. This result and the result that shows how the large firm benefits dramatically from a unilateral lowering of its cost factor suggest that an innovating small firm has an incentive to incur short-term losses to try and displace the large firm from its preferable market segment.

The large firm's position is also preferable in the case of the challenge of a third yet smaller firm entering the market. According to the model when a third firm enters, the profits become $\pi_L = (0.011742) \times \theta_u^4/\xi$, and $\pi_S = (0.000624) \times \theta_u^4/\xi$. The new firm will have profit of $(0.000027) \times \theta_u^4/\xi$. For the original two duopolists, the percentage fall in profit is 7.295% for the larger firm and 18.644% for the smaller firm. The profit of the middle firm in the new equilibrium is now about 5.31% of that of the larger firm. This is a fall from the value of $\pi_S/\pi_L = 6.06\%$ seen in the duopoly case.

This analysis has illustrated why a small firm using the strategy of producing a lower-quality product at a lower price has incentives to challenge the large firm. This is especially true if it is possible to unilaterally lower costs of production. The rewards to the small firm for increasing efficiency are relatively small within the framework of the shared market. Challenges from a third firm using the same strategy as the small firm when it entered will have a larger proportional impact on the small firm in the duopoly. These results depend a great deal on the assumptions of the model as discussed in the conclusion.

7. Conclusion

This paper has addressed concerns of a small firm that is planning to enter a market dominated by a single seller. Previous research supports the proposition that the small firm can enter if it chooses to produce a product of lower quality. Under a given set of assumptions, the model here has solved for the equilibrium values for prices, qualities, quantities, and profits; and it has demonstrated how they will change given shifts in exogenous and endogenous parameters.

The exogenous parameters considered here are the quality of a free variety of the good, a variable-cost factor and the size of the market. These parameters entirely determine the quality levels, price levels and profits of the two firms. Given the assumed cost function, changes in the free quality give: $\Delta q_F = \Delta q_S = \Delta q_L$. The quality of the large firm is more sensitive than that of the small firm, however, to changes in market size and cost. These results may prove important if future work expands the model to include adjustment costs.

Interestingly enough, in this model, the free quality level does not affect the price of either the large or small firm's product. The free quality level does not affect profit either. However, in absolute terms, the sensitivity of the large firm's price to exogenous changes in market size or costs is greater than that of the small firm. Referring to costs as exogenous presumes that they are determined by the general level of technology available to both firms. The level of quality of the free good could reflect this exogenous aspect of costs as well.

This paper has also considered a case where one type of cost is endogenous. More specifically each firm can innovate and lower its own cost factor unilaterally. The analysis focused on the effect of the small firm unilaterally lowering the value denoted ξ_S in the cost function for the small firm: $\xi_S \times (q_S - q_F)^2$. The gains to the small firm are fairly small. In fact, in this model, the small firm is better off sharing innovations that increase efficiency with the large firm. The analysis went on to show that the large firm is in a much better position to benefit from unilateral increases in efficiency. The implication is that if the small firm can lower its costs, then it should consider challenging the larger firm for the dominant position.

Finally, the small firm suffers more when a third and smaller firm enters the market. The now intermediate firm loses a larger percentage of its profit than the largest firm. This is another reason why the small firm may choose to change the nature of the competition at some point.

Despite the results of this model, many markets support more than one quality of a good. Less-expensive, low-quality goods are often very successful. One of the most important factors for this success is probably the distribution of tastes and income. The model in this paper assumes that income is not a factor and tastes are uniformly distributed. Thus, the higher quality good never becomes too expensive for purchase, and there is not a disproportionately large group of consumers looking for a basic type of the product to consume. In contrast, for example, low-price and quality automobile firms have faired quite well for most of this century. Christensen (1997) explores several recent cases where the strategy of entering a market at the low-end of the quality spectrum has been quite successful. The primary point of this paper has been to illustrate a case and, more generally, a methodology for exploring the considerations for a small firm with a low-quality product. Firms planning to enter a market can look to this model and see how the targeted market is similar to the model before planning the strategy for entry.

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Notes

[1] The first derivative is $d\pi/dq_L = (\theta_u^2/4) - 2 \times \xi \times (q_L - q_F)$. The second-order condition is satisfied: $d^2\pi/dq_L^2 = -2 \times \xi$.

[2] There are distributions for which θ^* is not unique. One example is $\phi(\theta) = 1 - K/\theta$; θ: $[K, +\infty)$, where K is a positive constant.

[3] Bertrand (1883) demonstrates that if firms compete directly with price, then prices will be driven down to marginal costs. This is referred to as "Bertrand death."

[4] This is a widely used assumption, see Tirole (1988) and Jehiel (1992).

[5] Anderson, DePalma, and Thisse (1992) recognize that "A necessary and sufficient condition for an equilibrium with both firms serving the market is $\theta_u > 2 \times \underline{\theta}$; that is, consumers must be sufficiently heterogeneous in terms of their marginal willingness to pay for quality." [p. 309] In the cases that follow, $\underline{\theta} = 0$; the condition is satisfied.

[6] The first-order conditions yield reaction functions in the plane defined by q_S and q_L. The first-order conditions give: $q_L = 5.25123 \times q_S - 4.25123 \times q_F$; therefore when equilibrium q_L and q_S exists, it is unique. The second-order conditions are satisfied:

$$\frac{d^2\pi_L}{dq_L^2} = \frac{-8 \times \theta_u^2 \times (\delta_{S,F} + 5 \times \delta_{L,F}) \times \delta_{S,F}^2}{\omega^4} > 0,$$

$$\frac{d^2\pi_S}{dq_S^2} = \frac{-2 \times \theta_u^2 \times (7 \times \delta_{S,F} + 8 \times \delta_{L,F}) \times \delta_{L,F}^2}{\omega^4} > 0.$$

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